Problem 1.1

line segment- consists of 2 points of a line and all the points between these 2 points

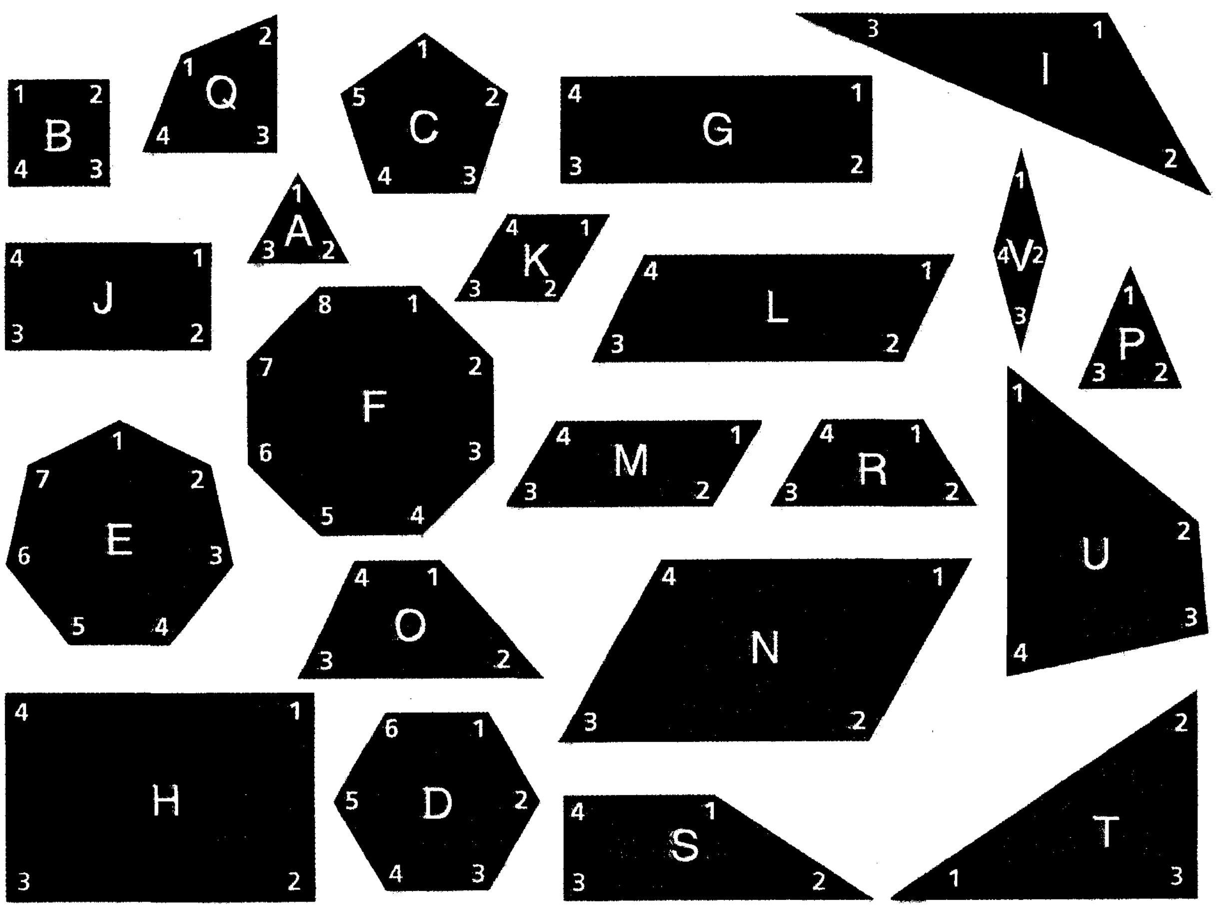
sides- the line segments that make up a polygon

vertices- the points where 2 sides of a polygon meet

Common Polygons

Number of sides and angles	Polygon Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon

Below are a variety of polygons. Many of these polygons have common properties



Part A

Sort the polygons in the Shapes Set into groups so that the polygons in each group have one or more properties in common. Describe the properties the polygons have in common and give the letters of the polygons in each group.

Part B

Take all the triangles and sort them into 2 or more groups. Describe the properties the triangles have in common and give the letters of the triangles in each group.

Part C

Take all the quadrilaterals and sort them into two or more groups. Describe the properties the quadrilaterals have in common and give the letters of the quadrilaterals in each group.

Part D

Rose put Shapes R, O and S into the same group. What properties do these polygons have in common?

Shape U would/would not belong to this group because ______________________________

____

Problem 1.2

Equilateral triangle- a triangle with 3 sides the same length

Isosceles triangle- a triangle with 2 sides the same length

Scalene triangle- a triangle with 0 sides the same length

Square- a quadrialteral with 4 sides the same length and 4 right angles

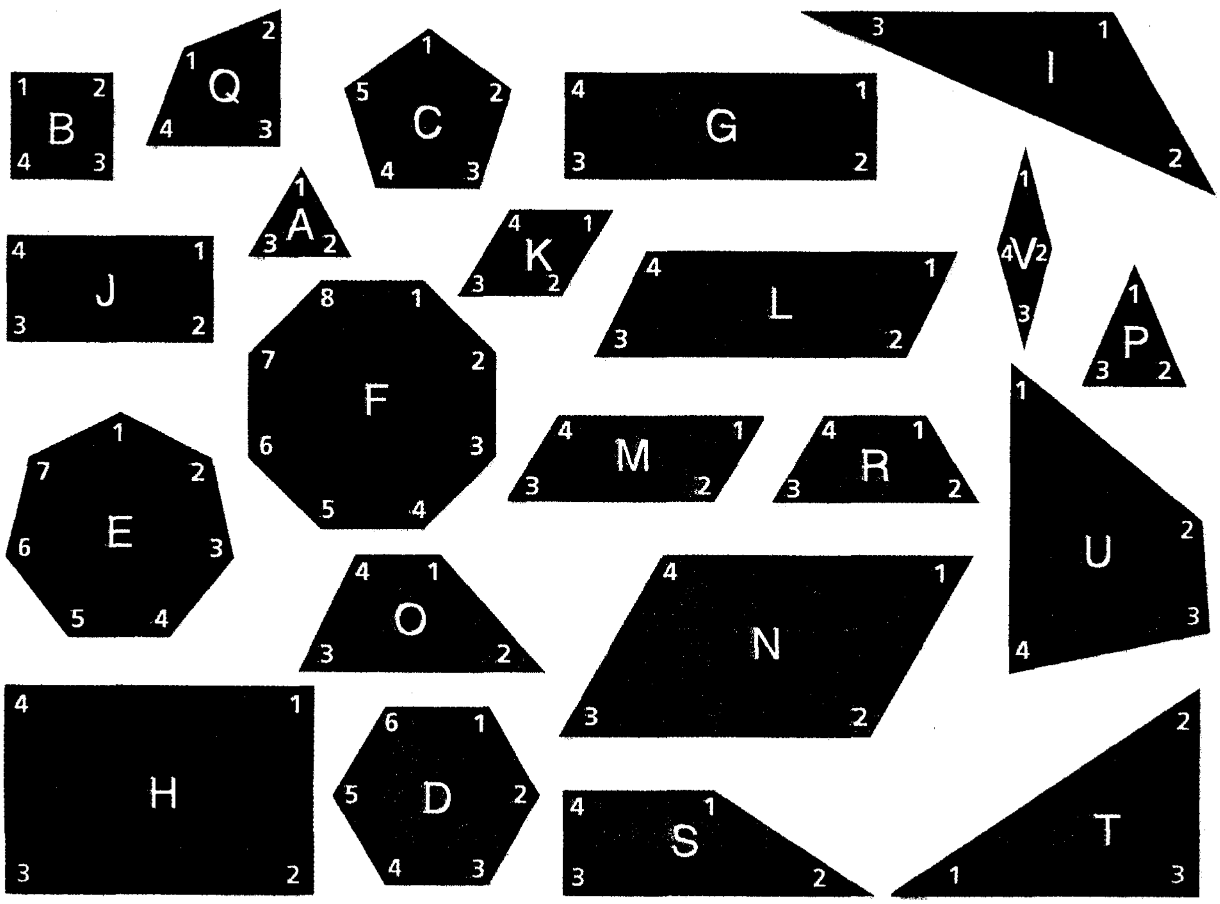
Rectangle- a quadrilateral with opposite sides the same length and 4 right angles

Parallelogram- a quadrilateral with opposite sides the same length and opposite angles of the same measure

Reflection symmetry- 2 halves that are mirror images of each other

Line of symmetry- the line that splits the shape into 2 halves that match exactly

Rotational symmetry- when you rotate a shape less than a full turn about its center point and it looks exactly as it did before



Shapes and Designs

Part A

Use the shapes on the other side. Look at the triangles.

1. Which triangles have reflection symmetry? Trace these triangles and draw all the lines of symmetry.

2. Which triangles have rotation symmetry?

3. Which triangles have no symmetries?

Part B

Look at the quadrilaterals.

1. Which quadrilaterals have reflection symmetry? Trace these quadrilaterals and draw all the lines of symmetry.

2. Which quadrilaterals have rotation symmetry?

3. Which quadrilaterals have no symmetries?

Part C

Look at the remaining polygons (the polygons that are not triangles or quadrilaterals). What is special about these shapes?

Part D

Find shapes with symmetry in your classroom. Sketch each shape and describe its symmetries.

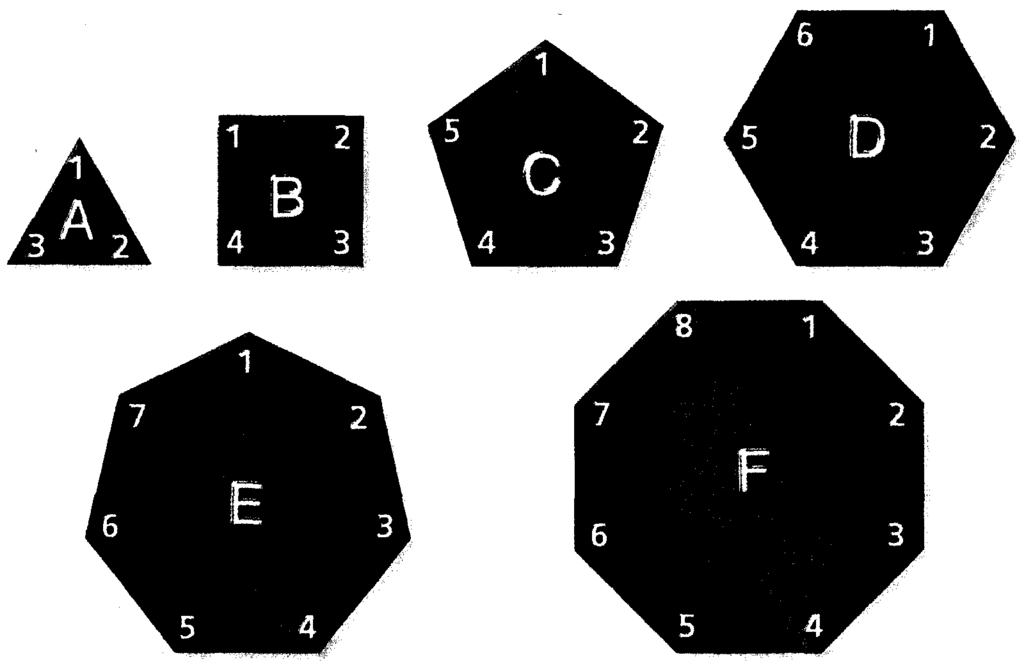
Problem 1.3

Regular polygon- a polygon in which all the sides are the same length and all the angles have the same measure.

Irregular polygon- all sides are *not* the same length or all the angles are *not* the same measure.

Tiling- means covering a flat surface with shapes that fit together without any gaps or overlaps.

Use the Shapes A-F below or cutouts of those shapes. As you work, try to figure out why some shapes cover a flat surface, while others do not.



Part A

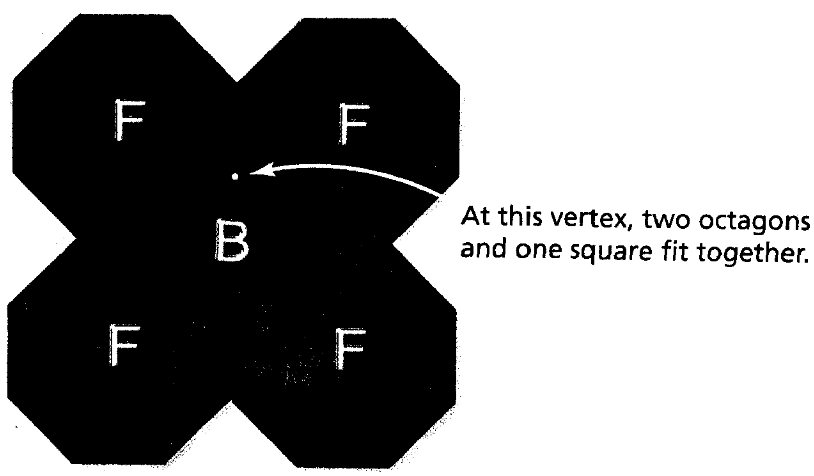
1. First, form tile patterns with several copies of the *same* polygon.

   Try each of the regular polygons. Sketch your tilings.

2. Which regular polygons fit together, without gaps or overlaps, to cover a flat surface?

Part B
1. Next, form tile patterns using combinations of 2 or more different shapes. Sketch your tiling.

Part C
The following tiling may be one that you found. Look at a point where the vertices of the polygons meet.



Shapes and Designs

1. Look back at each tiling you made. Find a point on the tiling where the vertices of the polygons meet.

2. Describe exactly which polygons fit around this point and the pattern of how they fit together.

3. Is this pattern the same for all other points where the vertices of the polygons meet in this tiling?

Problem 2.1

Rays- half lines 

Vertex- the point of the ray

Degrees- the unit used to measure the size of an angle

Right angle- has a measure of 90°

Part A

Sketch the angles made by these turns. For each sketch, include a curved arrow indicating the turn and label the angle with its degree measure.

1. One third of a right angle turn

2. Two thirds of a right angle turn

3. One quarter of a right angle turn

4. One and a half right angle turns

5. Two right angle turns

6. three right angle turns

Shapes and Designs

Part B

In #1-6, sketch an angle with *approximately* the given measure. For each sketch, include a curved arrow indicating the turn.

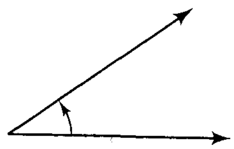
1. 20°
2. 70°
3. 150°
4. 180°
5. 270°
6. 360°

Shapes and Designs

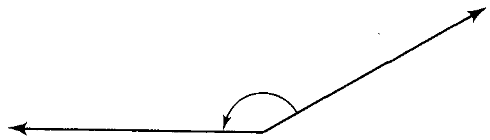
<u>Part C</u>

Estimate the measure of each angle.

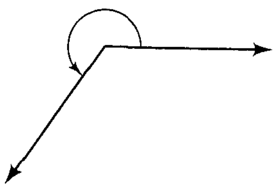
1.



2.



3.



Problem 2.2

Four in a Row Rules

Choose one of the circular grids, either with 30° intervals or 45° intervals.

- **Player A chooses a point where a circle and a grid line meet and says the coordinates of the point aloud.**
- **Player B checks that the coordinates Player A gave are correct. If they are, Player A marks the point with an X. If they are not, Player A does not get to mark a point.**
- **Player B chooses a point and says its coordinates. If the coordinates are correct, Player B marks the point with an O.**
- **Players continue to take turns, saying the coordinates of a point and then marking the point. The first player to get four marks in a row, either along a grid line or around a circle, wins the game.**

Part A

Play Four in a Row several times. Play games with the 30° grid and the 45° grid. Write down any winning strategies you discover.

Part B

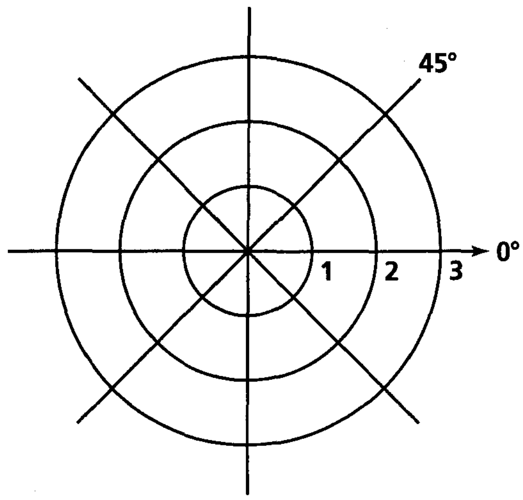
On one of the circular grids, label points A, B, and C that fit the following descriptions:

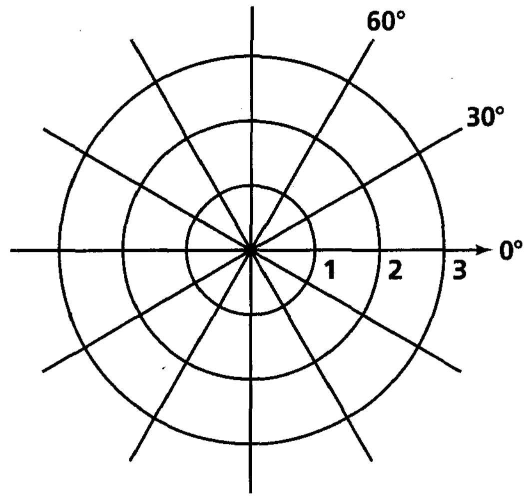
- The angle measure for point A is greater than 120°
- The angle measure for point B is equal to 0°
- The angle measure for point C is less than 90°

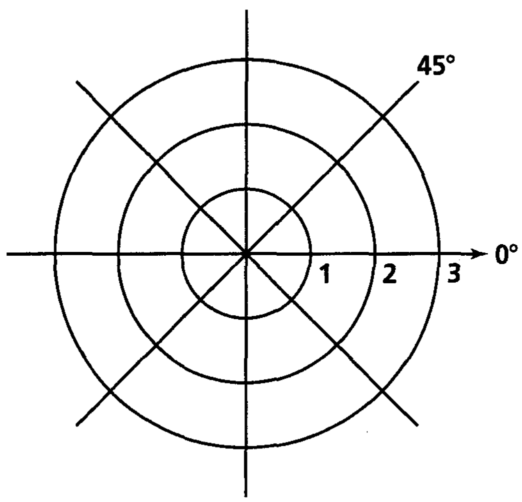
Name ______________________ Date ________ Class ________

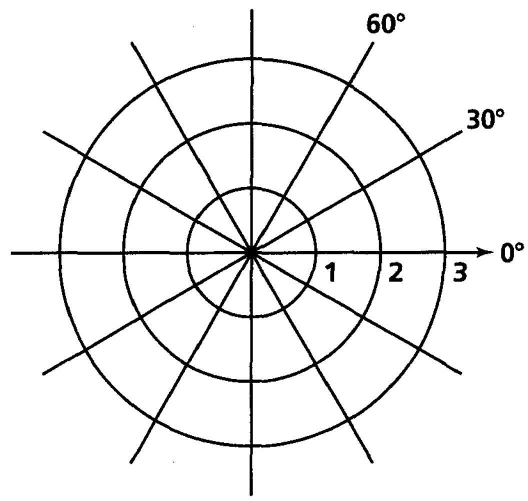
Labsheet 2.2

Four in a Row



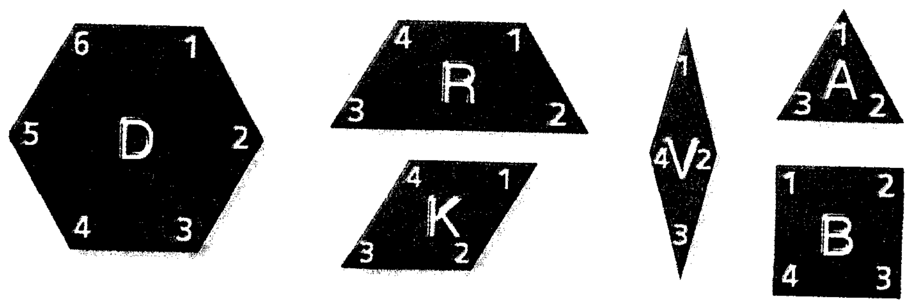






Problem 2.3

Part A
Use the shapes below to *estimate* the measure of each angle in the shapes. Label each angle with your estimate.



Part B
1. Use an angle ruler to *measure* each angle of the six shapes. On your drawing of the shapes, label each angle with its measure. Use a different color that you used on Part A.

2. How do your measurements compare with your estimates?

Shape D-

Shape R-

Shape K-

Shape V-

Shape A-

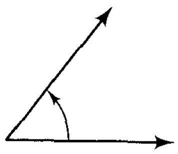
Shape B-

Shapes and Designs

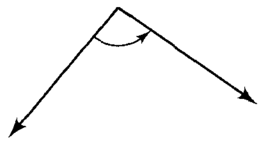
Part C

Use an angle ruler to find the measure of each angle.

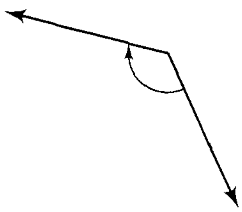
1.



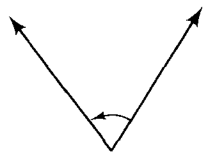
2.



3.



4.



Part D

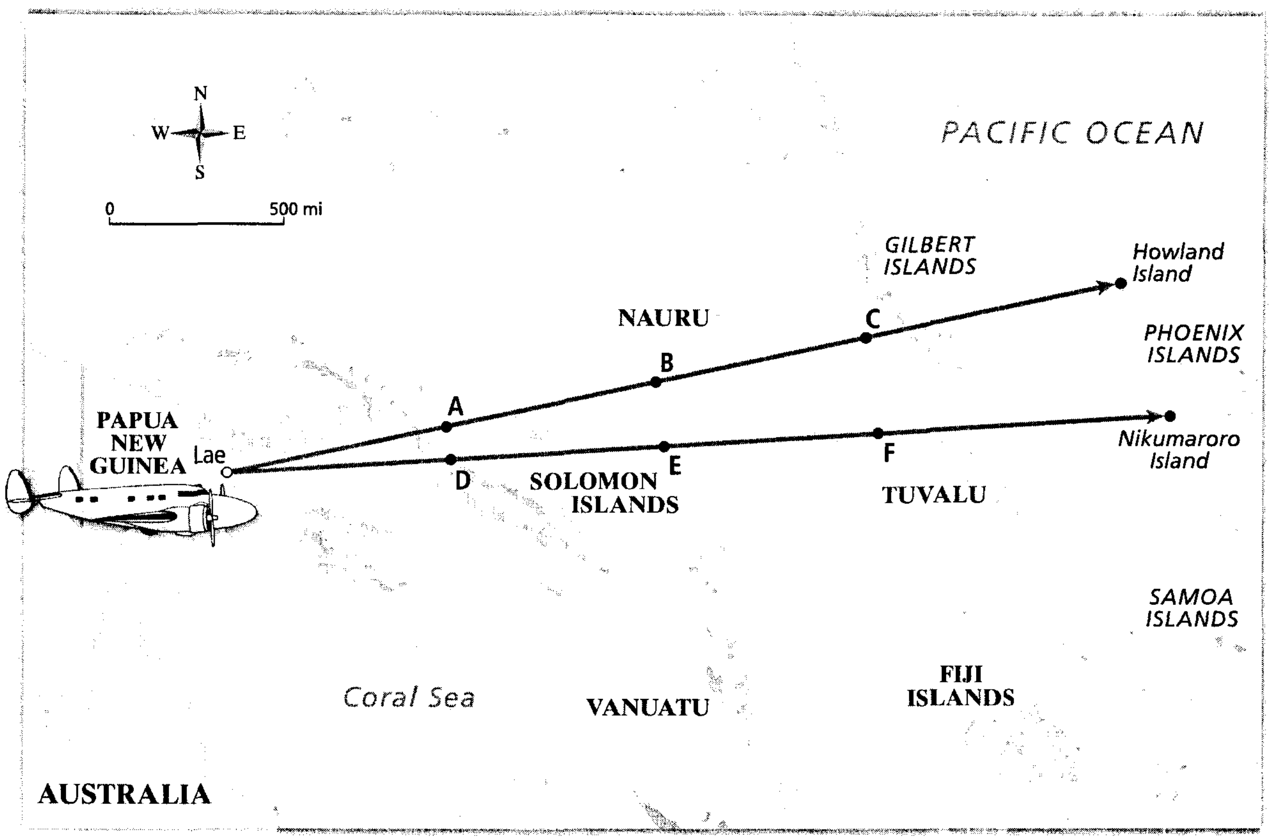
1. Draw an angle whose measure is less than the measure of any of the angles in Part C.

2. Draw an angle whose measure is greater than the measure of any of the angles in Part C.

Problem 2.4

The map below shows Lae, New Guinea; Howland Island (Earhart's intended destination); and Nikumaroro Island (the crash site).

Earhart Map



Part A

How many degrees off course was Earhart's crash site from her intended destination?

Part B

Suppose 2 planes fly along the paths formed by the rays of the angle indicated on the map. Both planes leave Lae, New Guinea, at the same time and fly at the same speed. Find the approximate distance in miles between the planes at each pair of points labeled on the map.

A and D B and E C and F

Part C

Amelia Earhart apparently flew several degrees south of her intended course. Suppose you start at Lae, New Guinea, and are trying to reach Howland, but you fly 20° south. Where might you land?

Problem 2.5

Parallel lines- lines that never meet

Transversal- a line that intersects 2 or more lines

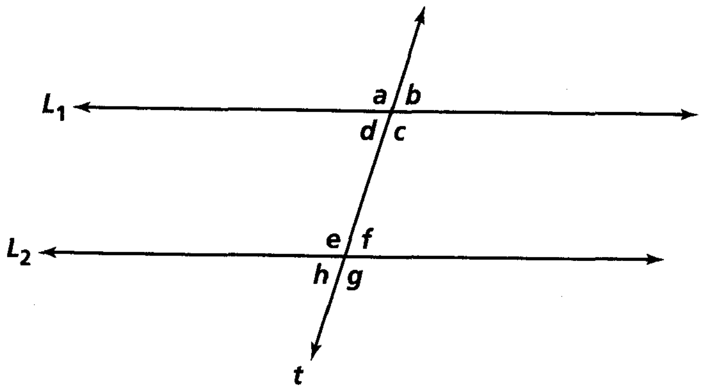


Figure 1

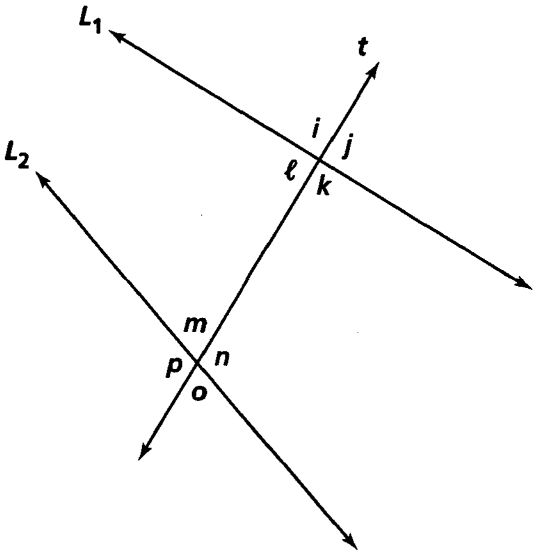


Figure 3

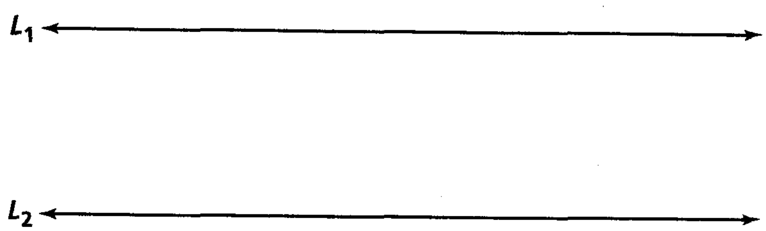


Figure 2

Part A

In Figure 1, lines L_1 and L_2 are parallel. They are intersected by a transversal *t*. Measure the angle labeled with small letters. What patterns do you observe among the angle measures?

Part B

In Figure 2, lines L_1 and L_2 are also parallel.

- Using the copy of Figure 2 on the other side of this sheet, draw a transversal *t* that intersects both lines.
- Measure the angles that are formed
- What patterns do you observe among the angle measures?

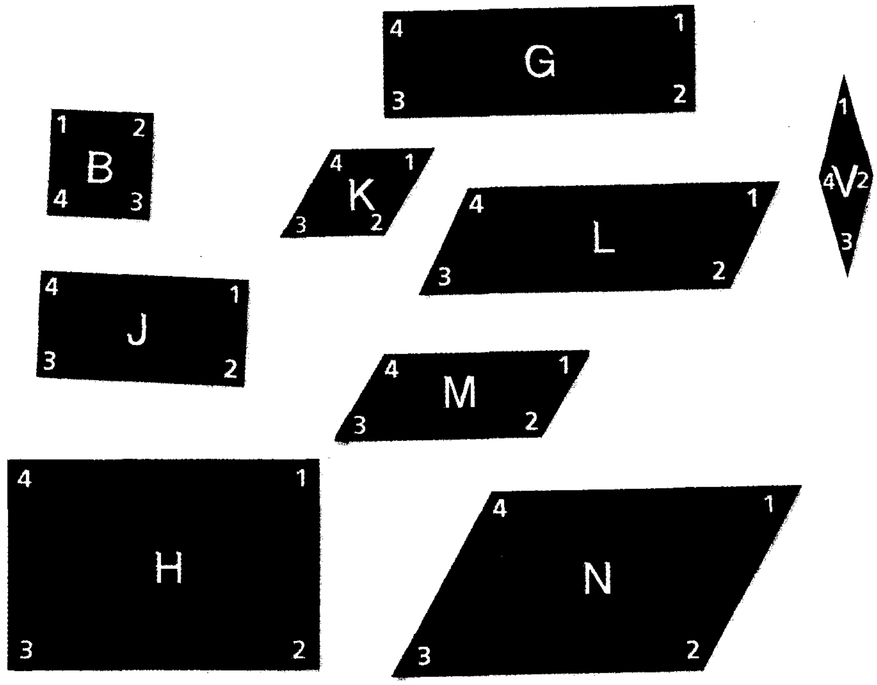
Part C

In Figure 3, lines L_1 and L_2 are *not* parallel.

- Measure the angles formed by the transversal intersecting lines L_1 and L_2
- Which patterns you observed in Figures 1 and 2 appears in Figure 3? Explain.

Part D

Make one or more conjectures about the measures of the angles formed when a transversal intersects 2 parallel lines.



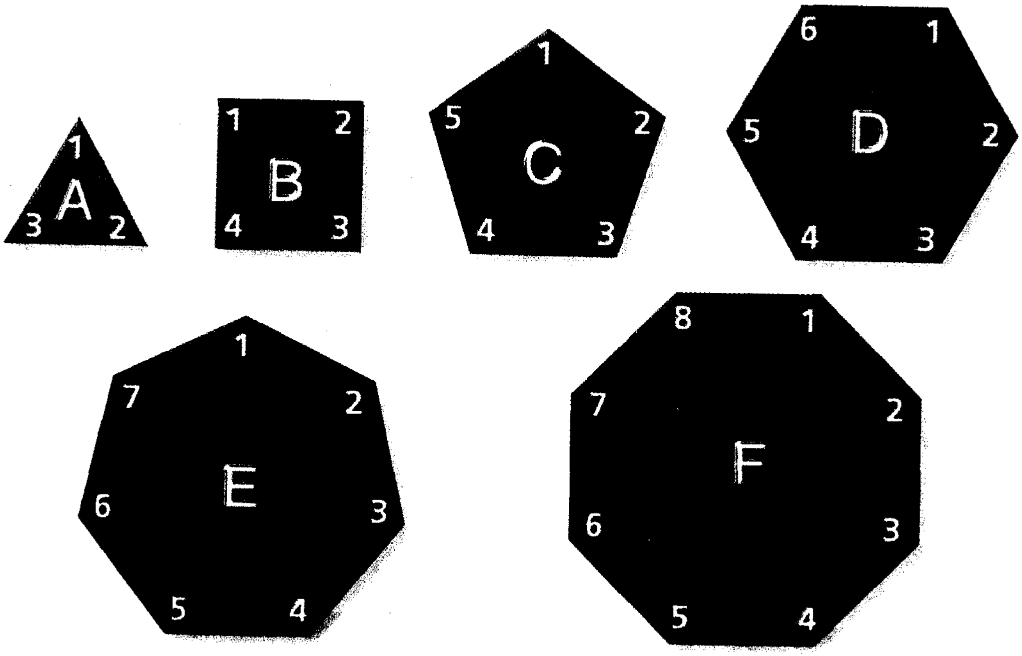
Part E

1. Explain why Shapes B, G, H, J, K, L, M, N and V in the Shapes Set are called parallelograms.

2. Trace 2 of the parallelograms. Are any of the lines in the parallelograms transversals? If so, which lines are the transversals?

3. Based on the conjectures that you have made in this problem, what do you think is true about the angle measures of a parallelogram? Check your ideas by choosing a parallelogram from the Shapes Set and measuring its angles.

Problem 3.1



Part A

1. In Problem 2.3, you measured the angles of some regular polygons- triangles, squares and hexagons. Record the number of sides, the angle measures, and the angle sum of a triangle, square, and hexagon in the table below.

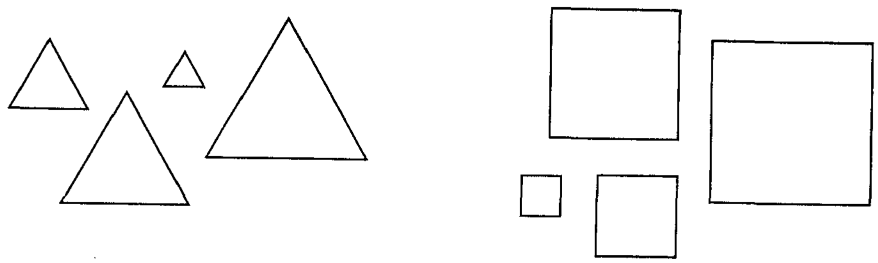
Polygon	Number of sides	Measure of angle	Angle Sum
Triangle			
Square			
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

2. Measure an angle of the regular pentagon and regular octagon from your Shapes Set. Record the measures of the angles and the angle sums in your table. What patterns do you see?

3. Use your patterns to fill in the table for a regular polygon with 7, 9 and 10 sides.

Part B

Below are two sets of regular polygons of different sizes. Do the same patterns relating the number of sides, the measures of the angles, and the angle sums apply to these shapes? Explain.



Part C

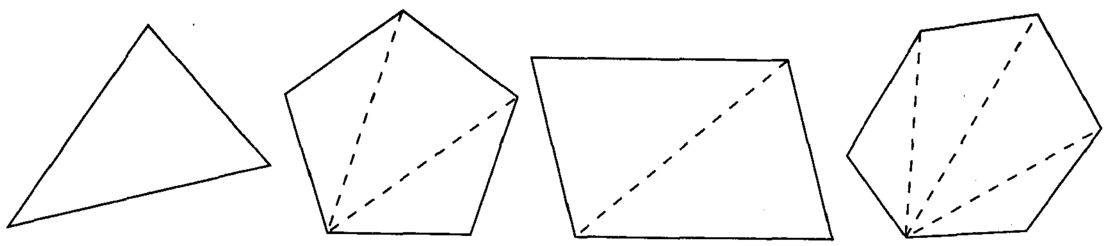
Describe how you could find the angle sum of a regular polygon that has *N* sides.

Problem 3.2

Tia and Cody claim that the angle sum of any polygon is the same as the angle sum of a regular polygon with the same number of sides. They use diagrams to illustrate their reasoning.

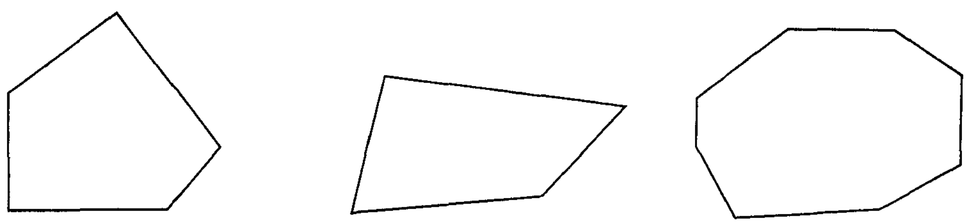
Part A

Tia divides polygons into triangles by drawing all the *diagonals* of the polygons form one vertex, as in the diagrams below:



1. Study Tia's drawings. How can you use Tia's method to find the angle sum of each polygon?

2. Use Tia's method on the polygons below to find the angle sum of each polygon.

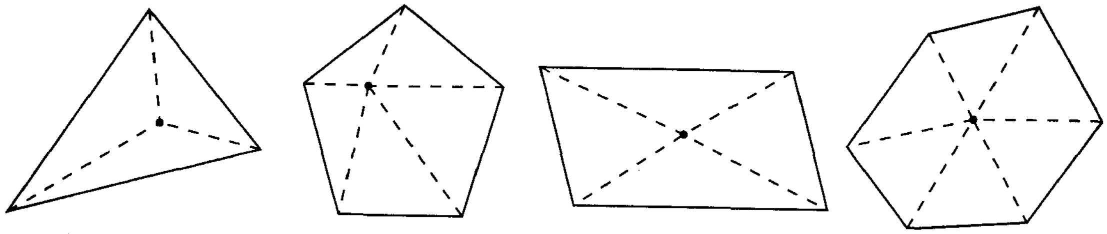


3. Does Tia's method work for any polygon? Explain.

Part B

Cody also discovered a method for finding the angle sum of any polygon. He starts by drawing line segments from a point within the polygon to each vertex.

1. Study Cody's drawings. How can you use Cody's method to find the angle sum of each polygon.



2. Use Cody's method on the shapes below to find the angle sum of each polygon.

3. Does Cody's method work for any polygon? Explain.

Part C

In Problem 3.1, you found a pattern relating the number of sides of a regular polygon to the angle sum. Does the same pattern hold for any polygon? Explain.

Problem 3.3

Part A

In Problem 1.3, you explored tilings made from a single type of regular polygon. You found that only equilateral triangles, squares, and regular hexagons could be used to tile a surface.

1. For each of these shapes, make a tiling and sketch the results.

Triangle

Squares

Hexagons

2. In each case, explain why copies of the shape fit neatly around a point.

Triangle

Squares

Hexagons

Part B

In Problem 1.3, you also found that regular pentagons, regular heptagons, and regular octagons could not be used to tile a surface. Explain why copies of these polygons do not fit neatly around a point.

Part C

1. Find tilings using combinations of 2 or more shapes from your Shapes Set. Sketch your results.

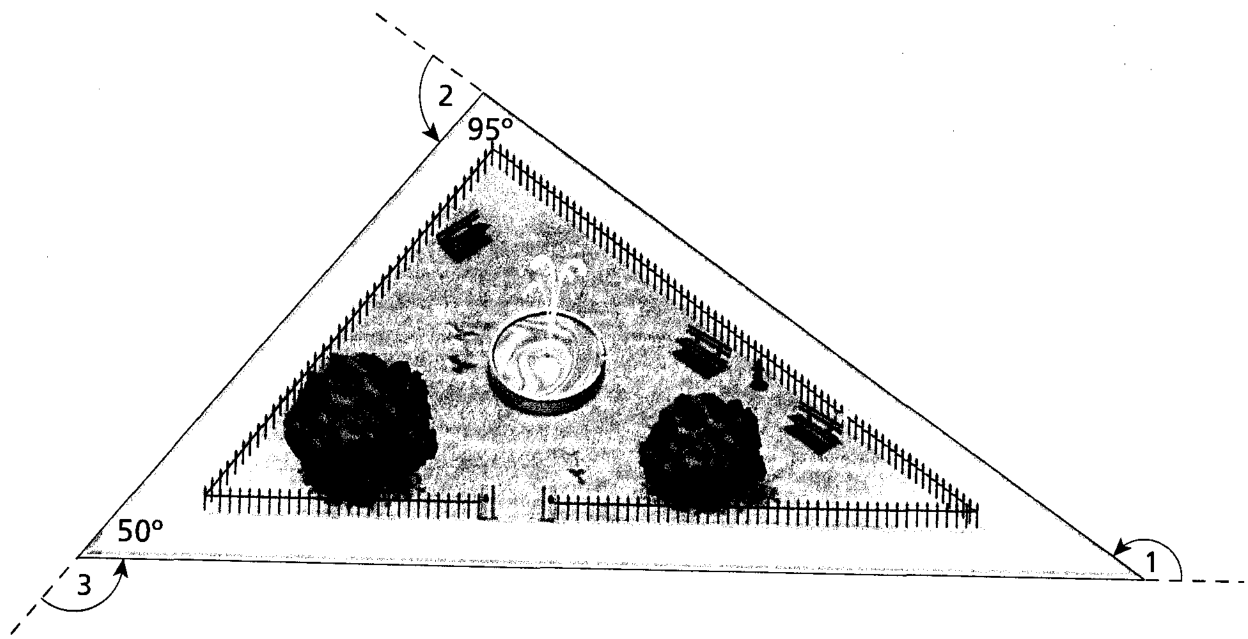
2. What do you observe about the angles that meet at a point on the tiling.

Problem 3.4

Interior angle- an angle inside a polygon

Exterior angle- the angle formed outside a polygon by extending the sides of the polygon

A skateboarder is skating on a triangular path around a park. In the diagram below, each segment of the path has been extended to show the angle of turn the in-line skater makes as she turns the corner. Each of these angles is an exterior angle of the triangle.



Part A

1. What are the measures of the interior angles of the triangle?

2. What is the measure of angle 1?

3. What is the measure of angle 2? Angle 3?

Part B

Suppose the skateboarder skates once around the park counterclockwise, turning each corner exactly one. What is the sum of the angles through which she turns?

Part C

1. Draw another triangle and mark the exterior angles going in one direction around the triangle.

2. Measure the exterior angles and find the sum.

3. Compare the exterior angle sum of your triangle to the sm you found for the triangle in Part B.

4. Can you predict the exterior angle sum for another triangle?

Problem 4.1

Make a triangle using the steps below. Sketch and label your results.

Step 1 Roll 3 number cubes and record the sum. Do this 2 more times, so that you have 3 sums.

Step 2 Using polystrips, try to make a triangle with the 3 sums as side lengths. If you can build one triangle, try to build a different triangle with the same side lengths.

Repeat steps 1 and 2 to make several triangles.

Shapes and Designs

Part A

1. List each set of side lengths that did make a triangle.

2. List each set of side lengths that did not make a triangle.

3. What pattern do you see in each set that explains why some sets of numbers make a triangle and some do not?

4. Use your pattern to find two new sets of side lengths that will make a triangle. Then find 2 new sets of side lengths that will not make a triangle.

Part B

Can you make 2 different triangles from the same 3 side lengths?

Part C

Why do you think triangles are so useful in construction?

Problem 4.2

Part A

1. Use polystrips to build quadrilaterals with each of the following sets of numbers as side lengths. Try to build 2 or more different quadrilaterals using the same set of side lengths.

6, 10, 15, 15	3, 5, 10, 20
8, 8, 10, 10	12, 20, 6, 9

2. Choose your own sets of four numbers and try to build quadrilaterals with those numbers as side lengths.

______, ______, ______, ______   ______, ______, ______, ______

______, ______, ______, ______   ______, ______, ______, ______

Part B

Use your observations from Part A.

1. Is it possible to make a quadrilateral using any 4 side lengths?

   If not, how can you tell whether you can make a quadrilateral from 4 side lengths?

2. Can you make 2 or more different quadrilaterals from the same 4 side lengths?

3. What combinations of side lengths are needed to build rectangles?

   What combinations of side lengths are needed to build squares?

   What combinations of side lengths are needed to build parallelograms?

Part C

1. Use 4 polystrips to build a quadrilateral. Press on the sides or corners of your quadrilateral. What happens?

2. Use another polystrip to add a diagonal connecting a pair of opposite vertices. Now, press on the sides or corners of the quadrilateral. What happens? Explain.

Part D

1. Describe the similarities and differences between what you learned about building triangles in Problem 4.1 and building quadrilateral in this problem.

Similarities

Differences

2. Explain why triangles are used in building structures more often than quadrilaterals.

Problem 4.3

Read the Quadrilateral Game Rules on page 74. Use the Quadrilateral Game board below.

The Quadrilateral Game

Row 6	A quadrilateral that is a square	**Add 1 point to your score and skip your turn**	A rectangle that is not a square	A quadrilateral with two obtuse angles	A quadrilateral with exactly one pair of parallel sides	A quadrilateral with one pair of opposite side lengths equal
Row 5	**Subtract 2 points from your score and skip your turn**	A quadrilateral that is not a rectangle	A quadrilateral with two pairs of consecutive angles that are equal	A quadrilateral with all four angles the same size	A quadrilateral with four lines of symmetry	A quadrilateral that is a rectangle
Row 4	A quadrilateral with no reflection or rotation symmetry	A quadrilateral with four right angles	**Skip a turn**	A quadrilateral with exactly one pair of consecutive side lengths that are equal	A quadrilateral with exactly one right angle	A quadrilateral with two 45° angles
Row 3	A quadrilateral with no angles equal	A quadrilateral with one pair of equal opposite angles	A quadrilateral with exactly one pair of opposite angles that are equal	**Add 2 points to your score and skip your turn**	A quadrilateral with no sides parallel	A quadrilateral with exactly two right angles
Row 2	A quadrilateral with both pairs of adjacent side lengths equal	A quadrilateral with two pairs of equal opposite angles	A quadrilateral with a diagonal that divides it into two identical shapes	A quadrilateral that is a rhombus	A quadrilateral with 180° rotation symmetry	**Subtract 1 point from your score and skip your turn**
Row 1	A quadrilateral with one diagonal that is a line of symmetry	A quadrilateral with no side lengths equal	A quadrilateral with exactly one angle greater than 180°	A parallelogram that is not a rectangle	**Add 3 points to your score and skip your turn**	A quadrilateral with two pairs of opposite side lengths equal
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6

Part A

Play the Quadrilateral Game. Keep a record of interesting strategies and difficult situations. Make notes about when you do not receive a point during a turn. Why did you not need to move any corners on those turns?

Part B

Write 2 new descriptions of quadrilaterals that you could include in the game grid.